Overturn dynamics of a small moon

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Abstract The paper considers the possibility of rotating a small moon by 180 degrees from a dynamic point of view using two rocket engines mounted on the surface of the moon. A thrust control law for the engines which allows to turn the moon over and then stabilize it relative to a new equilibrium position by parrying a gravity gradient torque is proposed. The dynamic aspects of such an experiment are discussed using the example of the Mars satellite Deimos.

Keywords: small moon, gravitational torque, 180-degree rotation, control torque

I. Introduction

It is difficult to imagine, but in the future, it is likely that a mission that requires the overturning of one of a small moon of the planets of the solar system [1-8] will be realized. The purpose of this paper is to show the possibility of a 180-degree rotation of a small moon from a mechanical point of view, and to propose this new problem of astrodynamics for discussion. The problem is studied in a planar formulation, when the small moon is acted upon only by a gravitational torque caused by the inequality of its main moments of inertia and by a control torque induced by two rocket engines installed on the surface of the moon. The paper discusses the dynamical aspects of such an experiment using the Deimos satellite of Mars as an example. The paper has the following structure. In Section 2, an equation is given for the planar attitude motion of a satellite (moon) under the action of the gravity gradient torque. Section 3 presents the control law of the rocket engines to perform the 180-degree rotation of the small moon and its further stabilization in the inverted position. Section 4 is devoted to the numerical simulation of the Deimos overturn. Finally, the conclusions are given in Section 5.

II. Mathematical model

Consider the plane motion of a moon in the gravitational field of a planet. A gravity gradient torque acting on the planet's moon is written as [9, p 370]

$$M_G = -\frac{3}{2} n^2 (A - C) \sin 2\varphi \tag{1}$$

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where φ is the angle of rotation of the moon relative to the local vertical, n is the mean orbital rate, A, B are the transverse moments of inertia and C is the axial moment of inertia of the moon. For a circular orbit and in the plane case of the attitude motion, the equation is as follows

$$B(\ddot{\theta} + \ddot{\varphi}) = -\frac{3}{2}n^2(A - C)\sin 2\varphi \tag{2}$$

where θ is the true anomaly. In the case of a circular orbit ($\dot{\theta} = n = const$) for the new independent variable $\theta = nt$, equation (2) takes the form

$$\varphi'' + \frac{3}{2}\sigma \sin 2\varphi = 0 \tag{3}$$

where $\varphi'' = \dfrac{d^2\varphi}{d\theta^2}$, $\sigma = \dfrac{A - C}{B} > 0$ is the inertial parameter. The inertia tensor of most moons is not a diagonal matrix of equal elements, i.e. a scalar matrix. Their principal moments of inertia are usually not equal to each other and due to the choice of coordinate axes the following rule holds

$$A > B > C \tag{4}$$

Eq. (3) has four equilibria. Two of them are stable

$$\varphi_{1,2} = 0, \pi \tag{5}$$

and the other two are unstable

$$\varphi_{3,4} = \pm\frac{\pi}{2} \tag{6}$$

If condition (4) is satisfied for planet's satellites, then there are two stable equilibrium positions (5). This results in the satellite's longitudinal axis Oz being oriented along the line connecting the centers of mass of the planet and its satellite. In this case, the moon is always facing one side to the planet.

III. Turning control law

This section is devoted to a rocket engine control law, which makes it possible to realize the turning of the small moon from one stable position ($\varphi_1 = 0$) to another ($\varphi_2 = \pi$) and its stabilization in the new stable position, i.e. to turn the previously invisible side of the moon towards the planet.

Suppose that each of the two rocket engines installed at opposite points on the moon surface (Fig. 1) produces a thrust equal to

$$P(\theta,\varphi,\varphi') = k\varphi' \operatorname{sgn}(\cos\varphi) + P_0 \delta \operatorname{sgn}(\varphi) \quad (7)$$

where $\varphi' = \dfrac{d\varphi}{d\theta}$, P is engine thrust, P_0 is thrust increment at the start of the turn, $k > 0$ is control factor, δ is the factor that ensures the initial swing of the moon over a period of one revolution of the moon around the planet

$$\delta = \begin{cases} 1, & \theta \le 2\pi \\ 0, & \theta > 2\pi \end{cases} \quad (8)$$

The first term in the control law **Ошибка! Источник ссылки не найден.** at $\varphi \in (-\frac{\pi}{2}, \frac{\pi}{2})$ implements further rotation relative to the initial stable position ($\varphi_1 = 0$), at $\varphi \in (\frac{\pi}{2}, \frac{3\pi}{2})$ it is provides stabilisation relative to the new stable equilibrium position ($\varphi_2 = \pi$). The second term in the control law **Ошибка! Источник ссылки не найден.** gives the initial rotation of the moon from the initial stable position ($\varphi_1 = 0$). Taking into account the control (7), Eq. (3) is written as

$$\varphi'' + \frac{3}{2}\sigma \sin 2\varphi = \frac{r}{Bn^2} 2P(\theta,\varphi,\varphi') \quad (9)$$

where r is the moon radius.

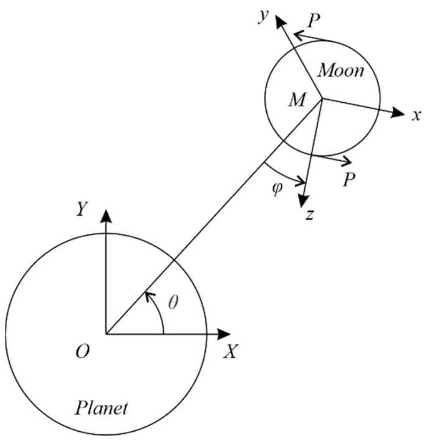


Fig. 1 Planet and Moon.

Note that Fig. 1 schematically depicts the moon in the shape of a sphere; in fact, no real moon has the shape of a sphere. It is important to interpret the radius of the moon in Eq. (9) correctly, it is actually the arm of the force P.

IV. Retrieval of the tether system into main spacecraft in low QSO

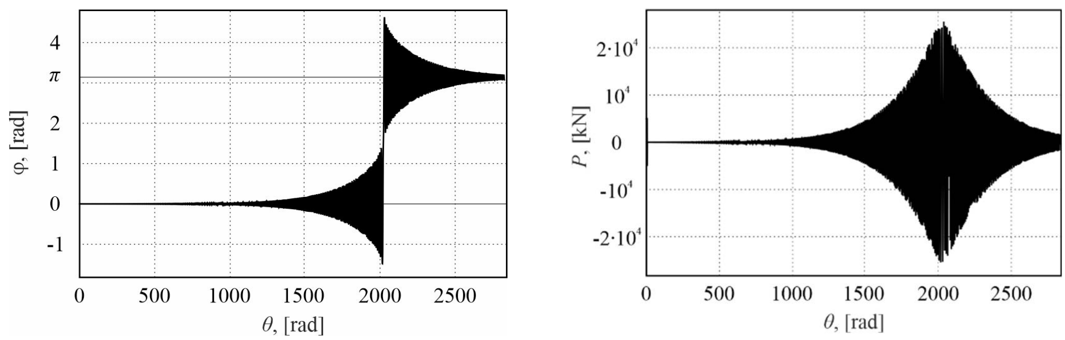
The Martian satellite Deimos is taken as an example to verify the proposed control law (7). It is clear that Deimos is not such a small satellite, but it is chosen because the principal moments of inertia are known for it [1,2]. The physical parameters of Deimos are shown in Table 1 [2]. The overturn manoeuvre is simulated by numerically integrating Eq. (9) for the following parameters of the control law (7):

$$k = 5 10^{7} N, \;\; P_{0} = 10^{7} N$$

Table 1 Major physics parameters of Deimos

Parameter	Value	Unit	Description
m	$1.48 \cdot 10^{15}$	kg	mass
r	6250	m	radius
A	$0.508\,mr^2$	$kg \cdot m^2$	moments of inertia
B	$0.461\,mr^2$	$kg \cdot m^2$	
C	$0.338\,mr^2$	$kg \cdot m^2$	

Fig. 2 shows (a) the overturning and stabilization of the moon relative to the new equilibrium position ($\varphi_2 = \pi$) and (b) the control force required for this manoeuvre.



(a) **(b)**

Fig. 2. (a) The angle rotation of the moon relative to the local vertical φ; (b) The control force P.

It should be noted that it would take about 600 Earth days to perform a 180-degree rotation of Deimos, and that the maximum thrust of an engine would have to be more than 20 million N. This is explained by the fact that Deimos has a large difference in the transverse moments of inertia $(A - C)$.

V.Conclusions

A new problem of astrodynamics is proposed for discussion, which may be widely discussed and realized in the future. Especially, it cannot be ruled out in the future that humanity will have a need to turn the moon reverse side in the Earth. Using the example of Deimos and a very simple control law, it is shown that the realization of its overturning requires large amounts of energy and time. If a moon has a small difference in transverse moments of inertia, then this problem would require less energy and time and might be easier to realize.

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